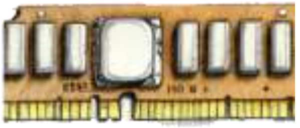
Proofs of Space



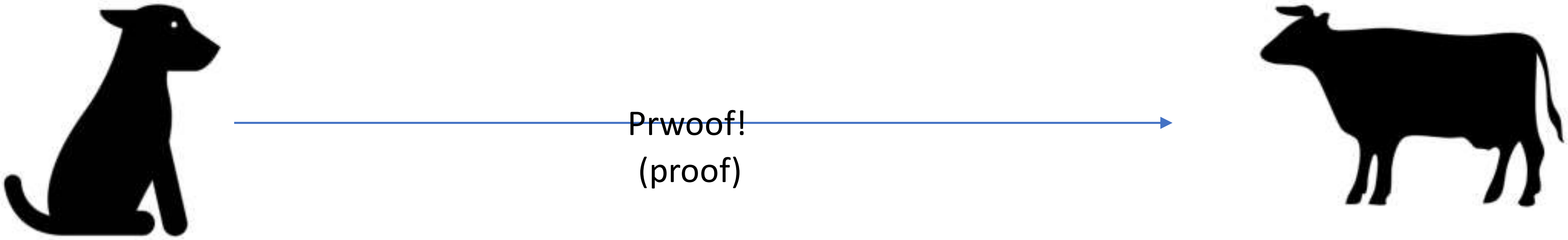
Matteo Campanelli

Protocol Labs

binarywhales.org matteo@protocol.ai

ASCrypto, Oct 3rd 2023

Yesterday’s Talk (by Arantxa)

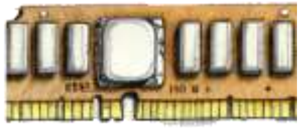


Prwoof!
(proof)

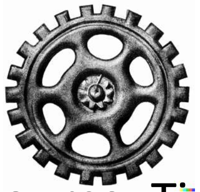
Witness w for a relation R(x,w)

Today: proofs yes; but proofs of ”resources”

Can we construct a proof which persuades we are *spending* a *certain (amount of) resource* X?



Resource=Space



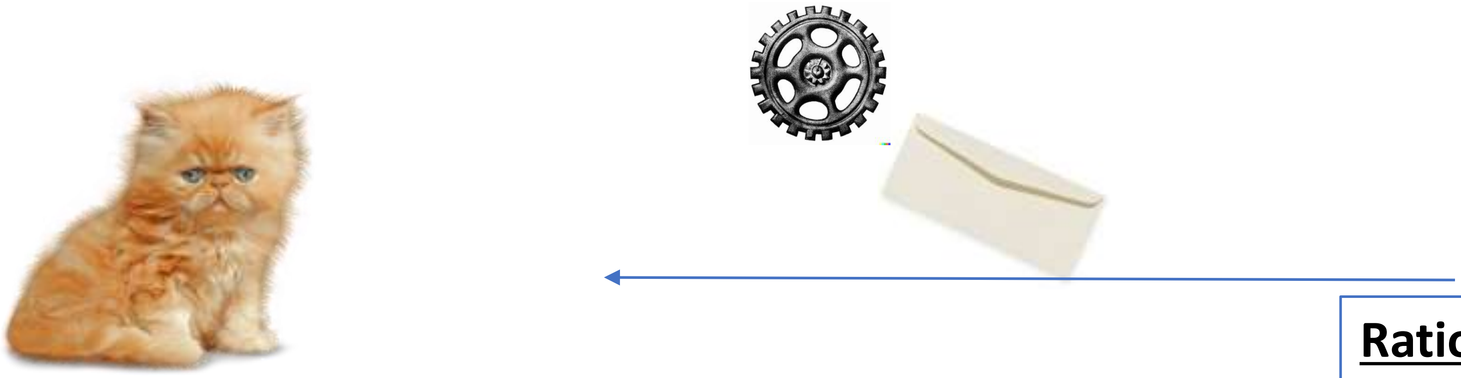
Resource=Time

Fighting Junk Mail (Spam)

Fighting Junk Mail Through *PoW* (Proofs of Work)

[DworkNaor (Crypto92)]

Proof of ”work” (or, CPU cycles)



I will not consider the email ”received” unless the PoW checks

Rationale:

- Spammers’ model: ”send cheaply a vast volume of mail”
- PoW now requires them to spend some amount per email (e.g. 0.0000005 cents)
- Not convenient any more in bulk
- **NB:** this is still OK for the honest (occasional) sender

A Simple Example of Proofs of Work

- **Hash-based:**
  - **Puzzle**: Sample random x (this is our challenge)
  - **Solution**: find r such that we find a certain amount of trailing zeroes

    H(x||r) = 0x(…*anything*…)000…000

  - **Intuition**: If there are M trailing zeroes this requires roughly 2^M on average
  - **To check the solution as a verifier:** receive r; check trailing zeroes in H(x||r)
    - (verifying is way faster than searching for the solution)

The single thing I want you to learn from this talk if you really *really* want to mentally leave right now

- PoW can save cat lives from spammers
  - (non-joke version: PoW is useful against spam)
- You can make a PoW by using some hash thing-y

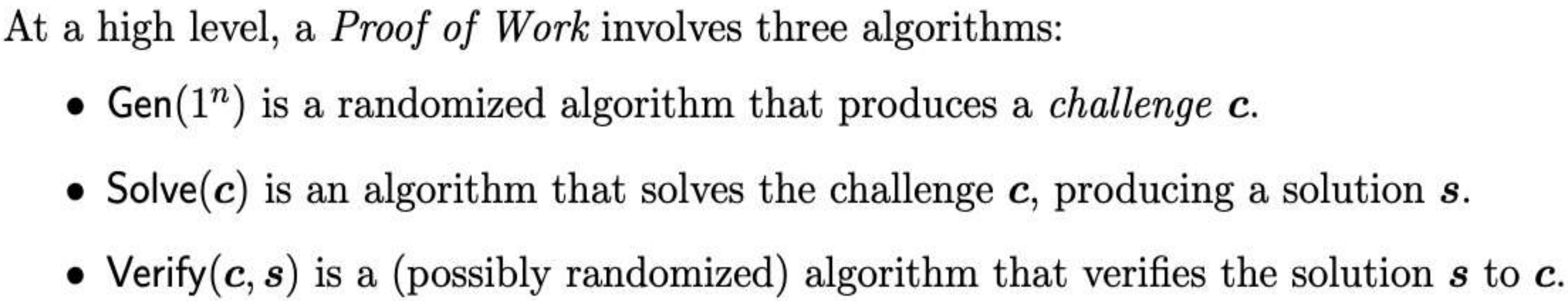
(assuming the hash output looks random enough)

Next on proofs of work (I lied: I would you like to take away a little bit more out of this talk)

- **Definition**: What does a definition for PoW look like?
- **Applications:** What are other applications for PoW?
- **Caveats**: what are limitations of applying Proofs of Work?

Defining PoW: Syntax

Proof of Work



At a high level, a *Proof of Work* involves three algorithms:

- $\mathsf{Gen}(1^n)$ is a randomized algorithm that produces a *challenge* $\boldsymbol{c}$.
- $\mathsf{Solve}(\boldsymbol{c})$ is an algorithm that solves the challenge $\boldsymbol{c}$, producing a solution $\boldsymbol{s}$.
- $\mathsf{Verify}(\boldsymbol{c}, \boldsymbol{s})$ is a (possibly randomized) algorithm that verifies the solution $\boldsymbol{s}$ to $\boldsymbol{c}$.

From Ball et al. 2017 ("Proofs of Useful Work") https://ia.cr/2017/203

Defining PoW: What about Security?

- This standard definition template won’t work:
  - “For all PPT * Adv … then Adv cannot win a certain game”

Gen(…) -> c

c

”bad” solution

Verify(c, bad_solution) =1

PPT ~

We are content with

* PPT: Probabilistic Polynomial Time

Defining PoW: Intuition for the right def

- Denote by T* := Time(Eval(c)) (honest solver’s time)
- For all adversaries running in time << T*
- This happens with very high probability:

Gen(…) -> c

c

”bad” solution

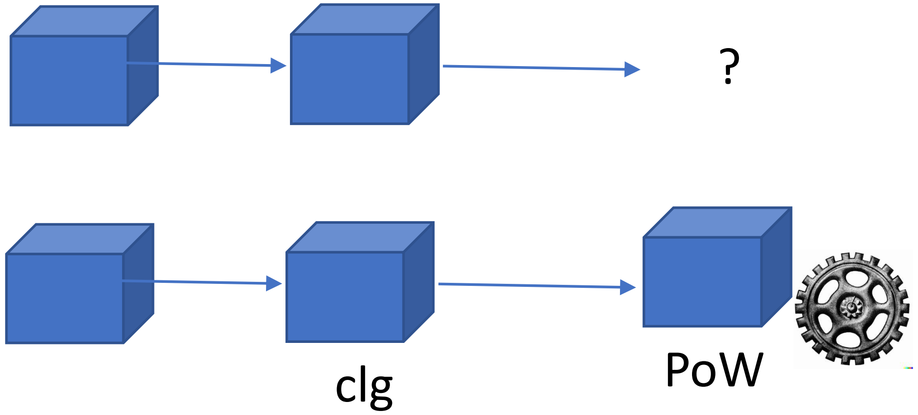
Verify(c, bad_solution) =0 (we do not accept)

Are we done?
One more thing:
amortization should be impossible.

If I ask you 100 challenges you should spend roughly 100 T* work.

Applications of Proofs of Work

- Fighting Spam/Denial-of-Service
- Bulletin Boards
  - (next block in a chain)



The second thing I want you to make sure you take from this talk in case you want to start snoozing now

When definining PoW:

- We do not use PPTs (as usual in cryptography); we specify the power of the adversary.
- We need to be careful about *amortization*.

Applications: denial of service in general; bullettin boards (e.g. Bitcoin)

Questions?

The <u>third</u> thing I want you to take from this talk before you start fantasizing about lunch

PoW have limitations.

Limitations/Caveats of PoW

- Waste
- The problem of ASICs*:
  - Or the ”Honest/Malicious gap”

*ASICs: Application-Specific Integrated Circuits

Issue: Waste of Energy in PoW

How to mitigate the problem of energy waste?

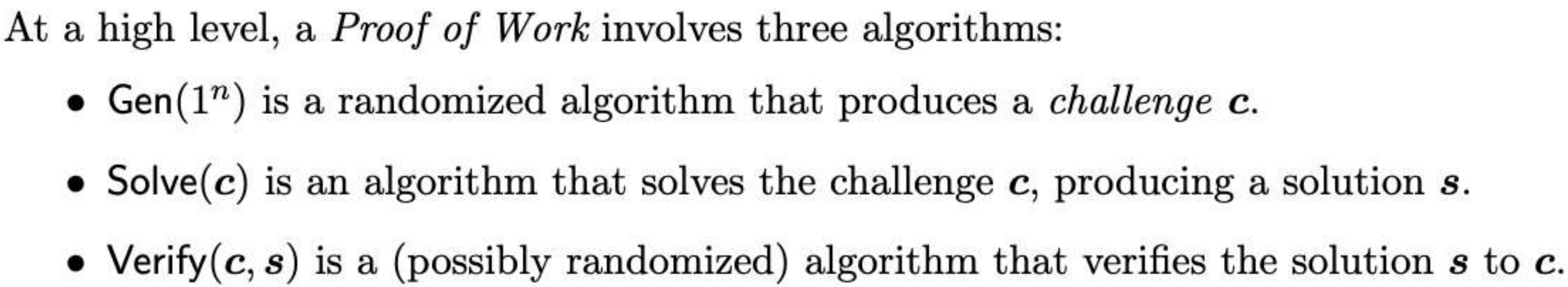
- Option 1:
  - Shifting to other resources (e.g. space)
- Option 2:
  - Actually making them ”useful”: using that grinding for “natural problems”

Mitigating Waste: Make PoW *Useful*

- The classic hash-based PoW does “nothing useful”
- Can we obtain a PoW that is “useful”?
- **Intuition**: “the grinding we are proving can be used for something else”
- **Example**: PrimeCoin (2013)
  - Introduces PoW based on search for prime numbers

A Syntax for Proofs of Useful Work

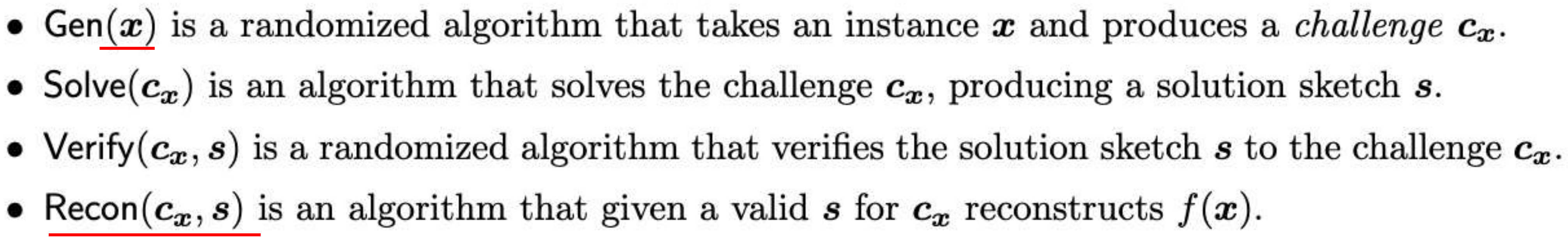
Standard Proof of Work



At a high level, a *Proof of Work* involves three algorithms:

- $\mathsf{Gen}(1^n)$ is a randomized algorithm that produces a *challenge* $\boldsymbol{c}$.
- $\mathsf{Solve}(\boldsymbol{c})$ is an algorithm that solves the challenge $\boldsymbol{c}$, producing a solution $\boldsymbol{s}$.
- $\mathsf{Verify}(\boldsymbol{c}, \boldsymbol{s})$ is a (possibly randomized) algorithm that verifies the solution $\boldsymbol{s}$ to $\boldsymbol{c}$.

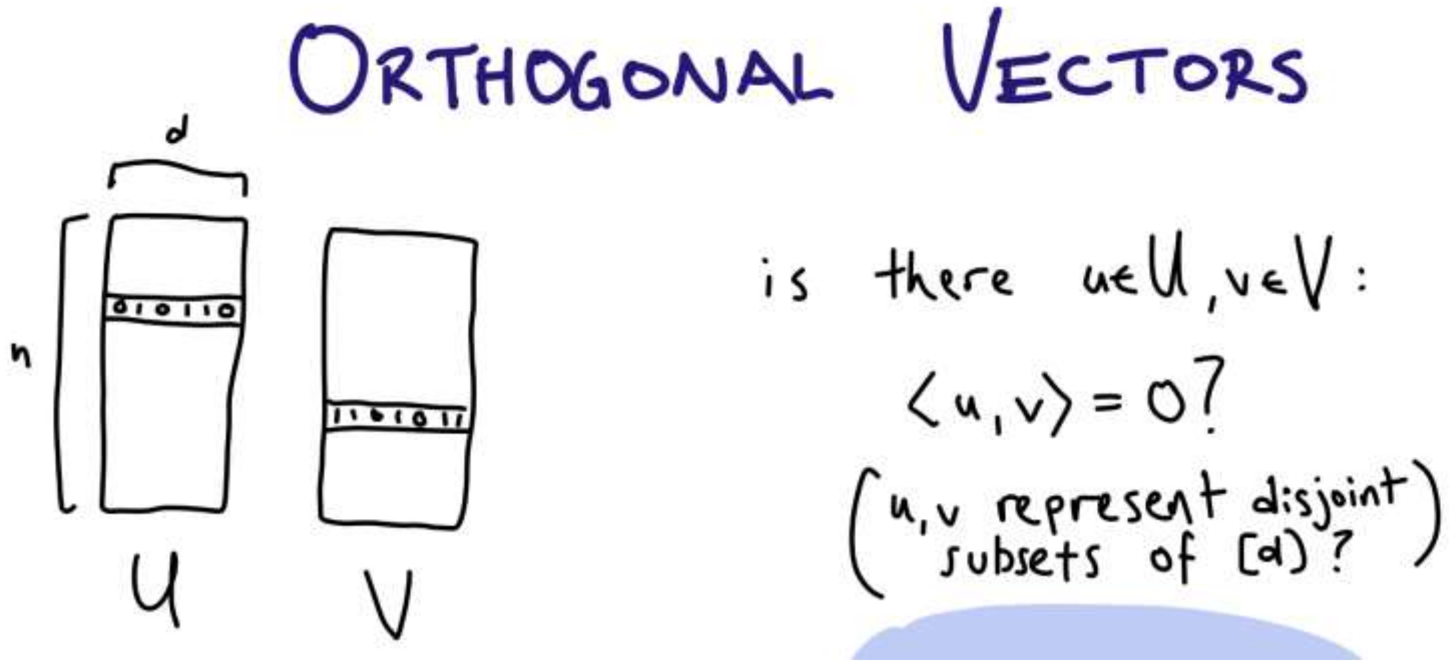
Proof of Useful Work



- $\mathsf{Gen}(\boldsymbol{x})$ is a randomized algorithm that takes an instance $\boldsymbol{x}$ and produces a *challenge* $\boldsymbol{c_x}$.
- $\mathsf{Solve}(\boldsymbol{c_x})$ is an algorithm that solves the challenge $\boldsymbol{c_x}$, producing a solution sketch $\boldsymbol{s}$.
- $\mathsf{Verify}(\boldsymbol{c_x}, \boldsymbol{s})$ is a randomized algorithm that verifies the solution sketch $\boldsymbol{s}$ to the challenge $\boldsymbol{c_x}$.
- $\mathsf{Recon}(\boldsymbol{c_x}, \boldsymbol{s})$ is an algorithm that given a valid $\boldsymbol{s}$ for $\boldsymbol{c_x}$ reconstructs $f(\boldsymbol{x})$.

From Ball et al. 2017 ("Proofs of Useful Work") https://ia.cr/2017/203

Examples of Proofs of Useful Work

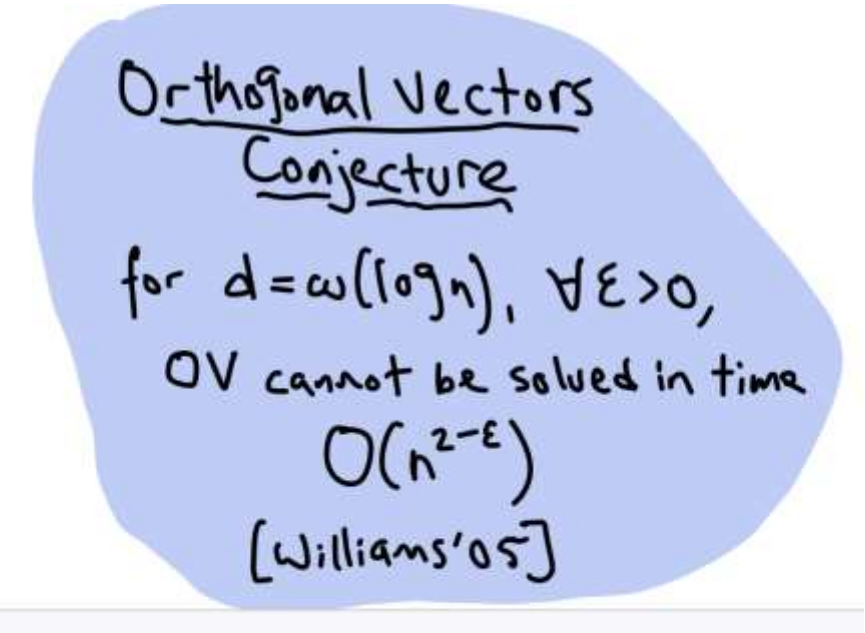


ORTHOGONAL VECTORS

is there $u \in U, v \in V$:

$\langle u, v \rangle = 0$?

(u, v represent disjoint subsets of [d]?)



Orthogonal Vectors Conjecture

for $d = \omega(\log n)$, $\forall \varepsilon > 0$,

OV cannot be solved in time

$O(n^{2-\varepsilon})$

[Williams'05]

Other natural examples: 3SUM, etc…

Credits to Marshal Ball for pics

Another Limitation: the ”ASIC” problem

Standard HW

Specialized HW for hashing (ASIC)

Incentive: making them pay 0.0000005 cents

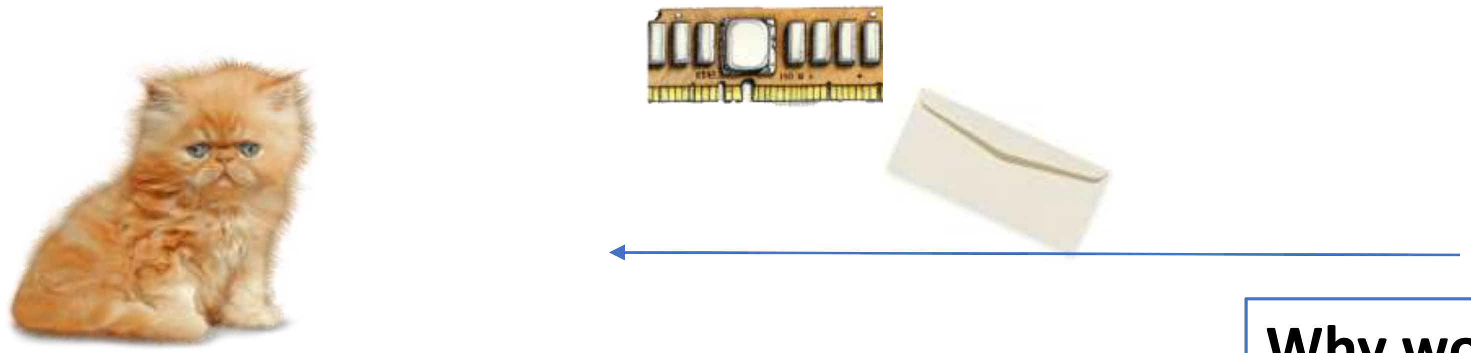
They will not pay the same anymore!

Last slides

- Proof of Work
  - Limitations:
    - can be wasteful, ASICs can raise the bar for honest parties
  - Mitigating waste
    - Useful Work
- What we will look at next:
  - Proof of Space
    - Same application realms
    - Our hope is to remove the limitations we saw:
      - Reduce the energy waste (we will talk about usefulness first)
      - Removing the ”ASIC problem”

Proofs of Space
(an intuition through our old friend: spam)

Proof of ”space”



The hoped guarantee: the adversary is must have used Y amount of space if proof checks

I will not consider the email ”received” unless the PoS checks

Why would this address PoW’s limitations?

- **Energy:** CPU vs RAM (or disk)
- **”ASIC”:** no equivalent for memory that can provide savings of orders of magnitude like ASICs did for CPU

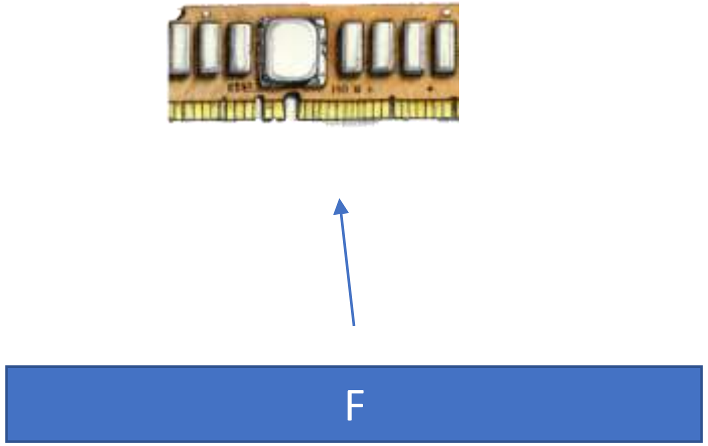
Applications: Making Proofs of Space *Useful*

- Applications of PoS (Proofs of Space)
  - Could be the same as before:
    - Figthing denial of service, etc.
    - Consensus for next block (Chia)
  - Or more (e.g., through usefulness)
    - By analogy, *usefulness* in PoW was: “I am using grinding for computation f(x)” (where f is some natural function)
    - “Natural usefulness in PoS”: let’s use PoS to **guarantee storage of useful files**
      - (E.g., data sets, Wikipedia, the web in general)
    - => System where we you can obtain cryptographic incentives of somebody using a certain amount of space AND using it that space for storing a specific file

Earlier during this talk:

Then

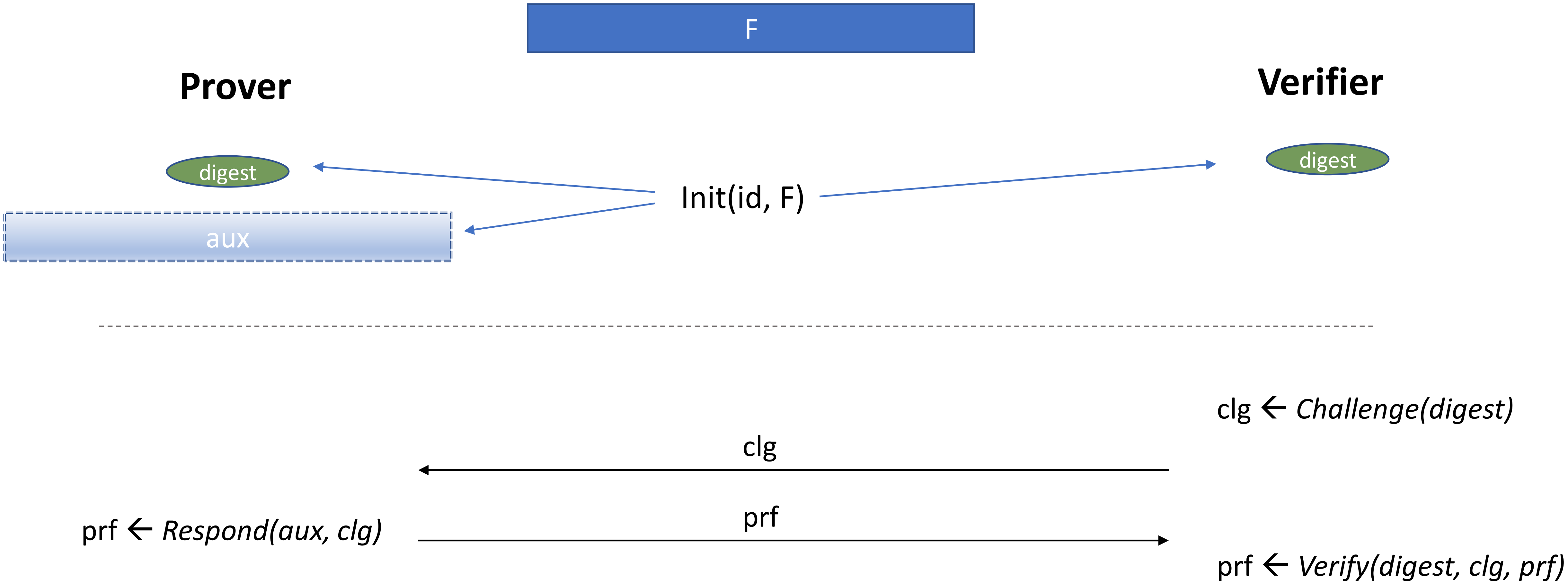
Now



Proof of Useful Space:
System where we you can obtain cryptographic incentives of somebody using a certain amount of space AND using it that space for storing a specific file

Next: I want to give a flavor of how one can define this

A syntax for Proofs of Useful Space



F

Prover

Verifier

digest

aux

Init(id, F)

digest

clg ← *Challenge(digest)*

clg

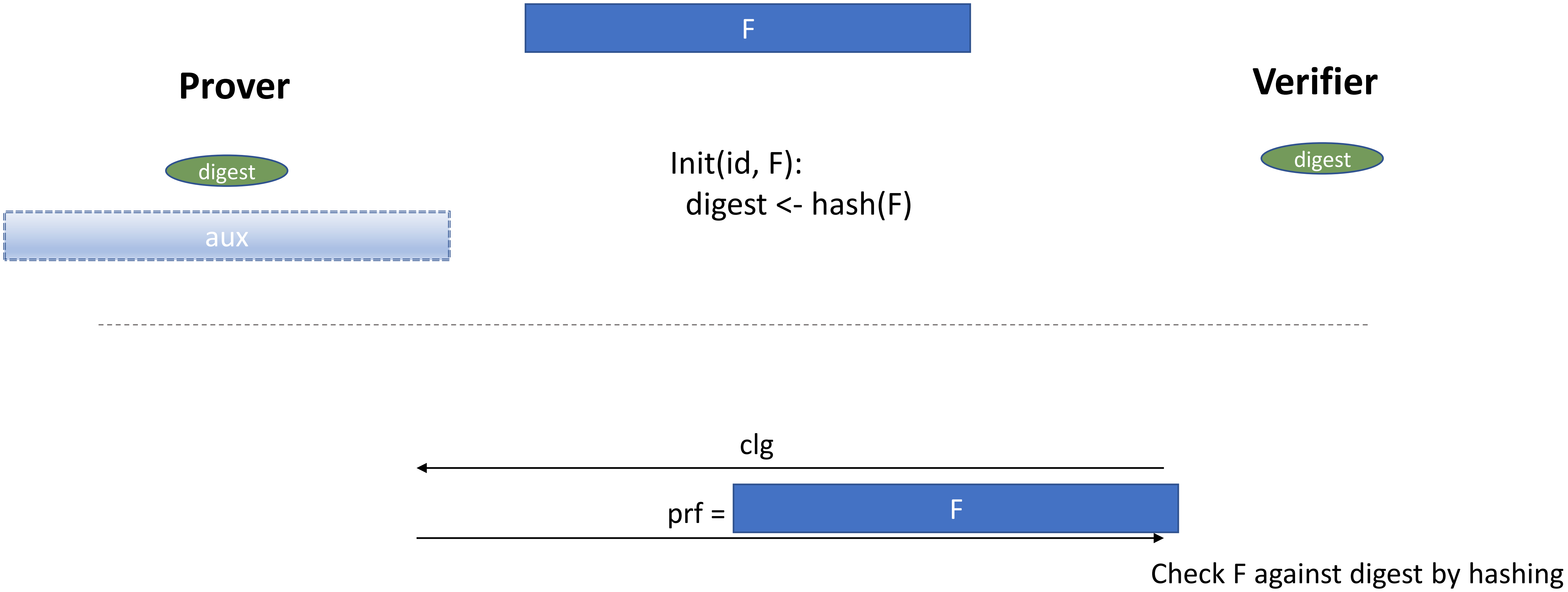
prf ← *Respond(aux, clg)*

prf

prf ← *Verify(digest, clg, prf)*

Security intuition: *“If prf checks then Prover is storing a certain amount of storage related to F”*

A non-solution

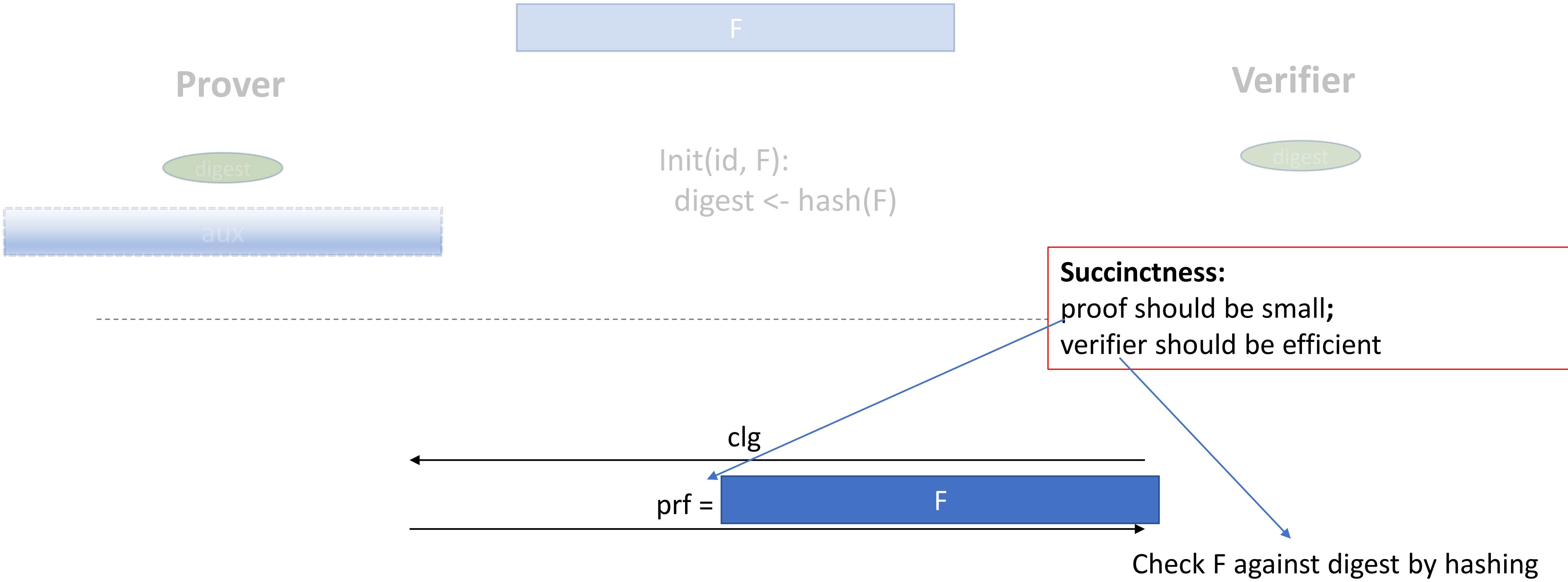


Init(id, F):
  digest <- hash(F)

Check F against digest by hashing

Security intuition: *“If prf checks then Prover is storing a certain amount of storage related to F”*

A non-solution—Issue 1: <u>succinctness</u>



F

Prover

Verifier

digest

aux

Init(id, F):
  digest <- hash(F)

digest

Succinctness:
proof should be small**;**
verifier should be efficient

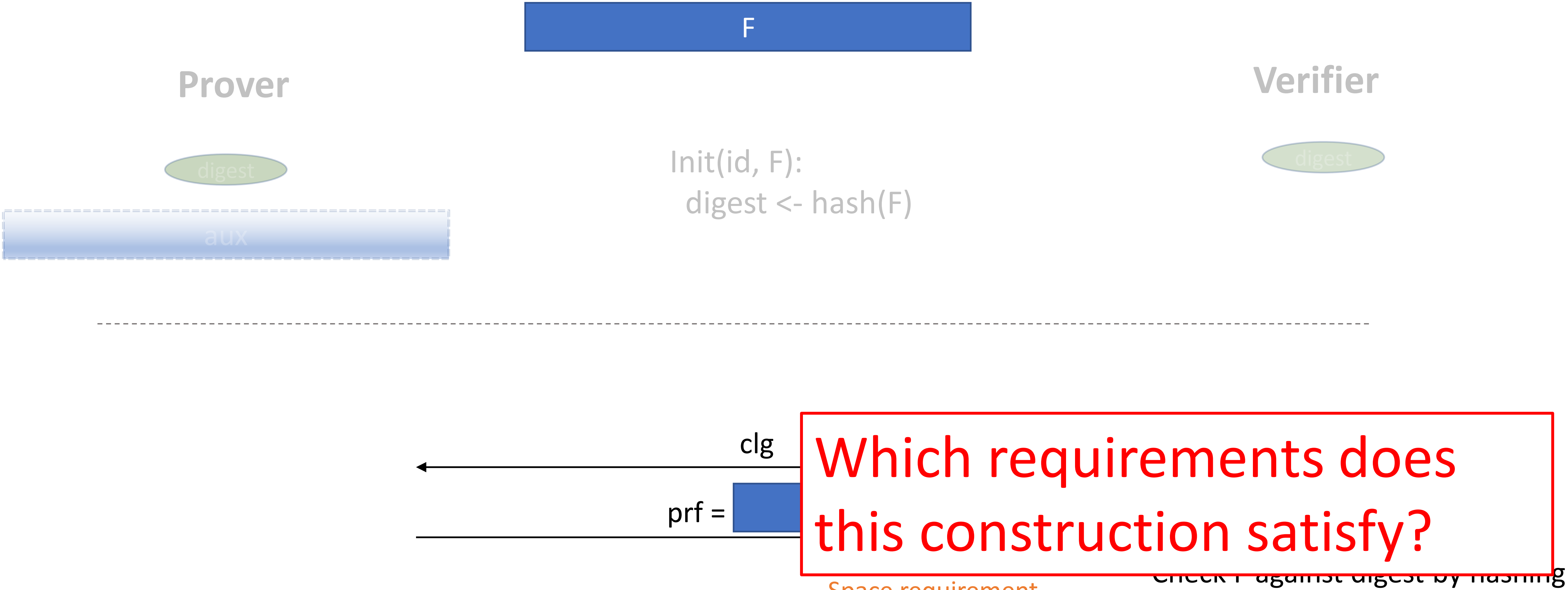
clg

prf = F

Check F against digest by hashing

Security intuition: *“If prf checks then Prover is storing a certain amount of storage related to F”*

A non-solution—Issue 2: space requirement



Init(id, F):
  digest <- hash(F)

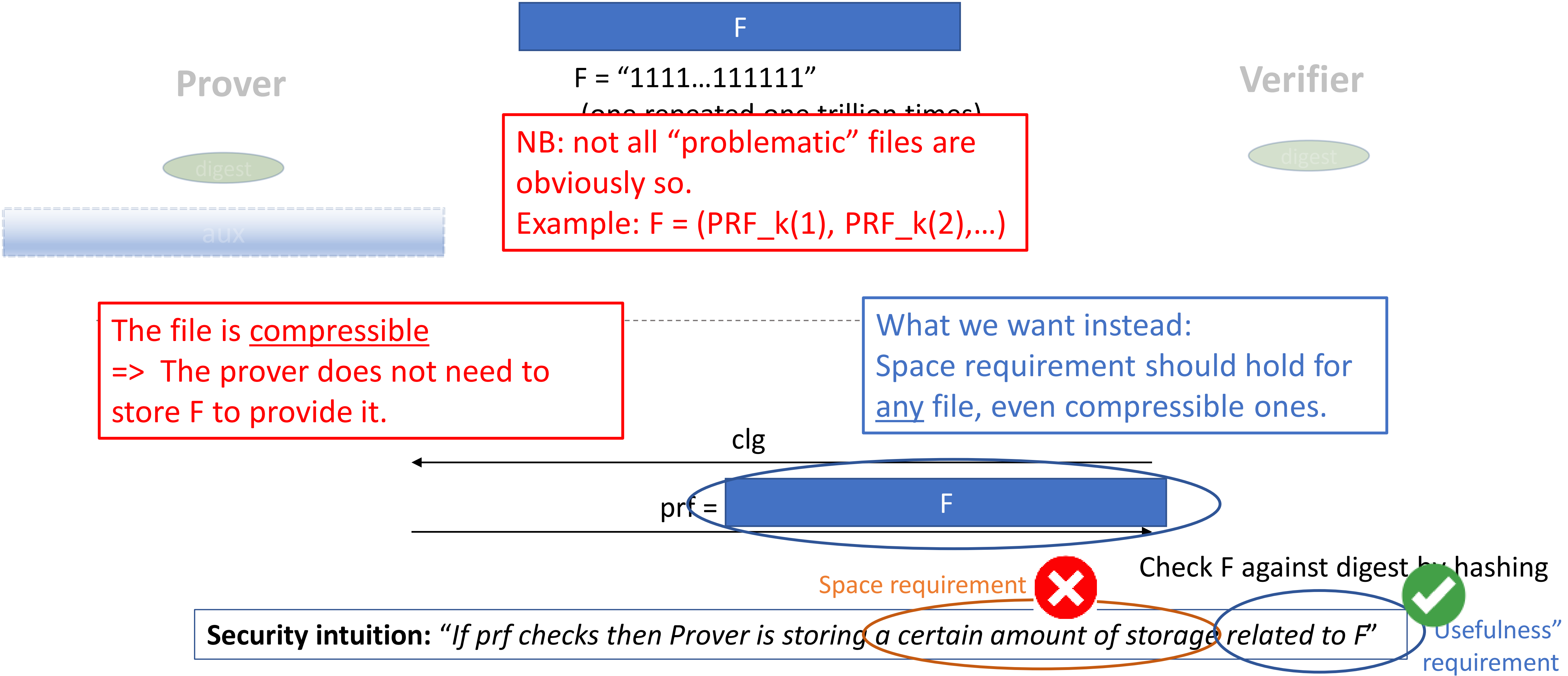
Which requirements does this construction satisfy?

Space requirement

Security intuition: *“If prf checks then Prover is storing a certain amount of storage related to F”*

”Usefulness” requirement

A non-solution—Issue 2: space requirement



F = “1111…111111”
(one repeated one trillion times)

NB: not all “problematic” files are obviously so.
Example: F = (PRF_k(1), PRF_k(2),…)

The file is compressible
=> The prover does not need to store F to provide it.

What we want instead:
Space requirement should hold for any file, even compressible ones.

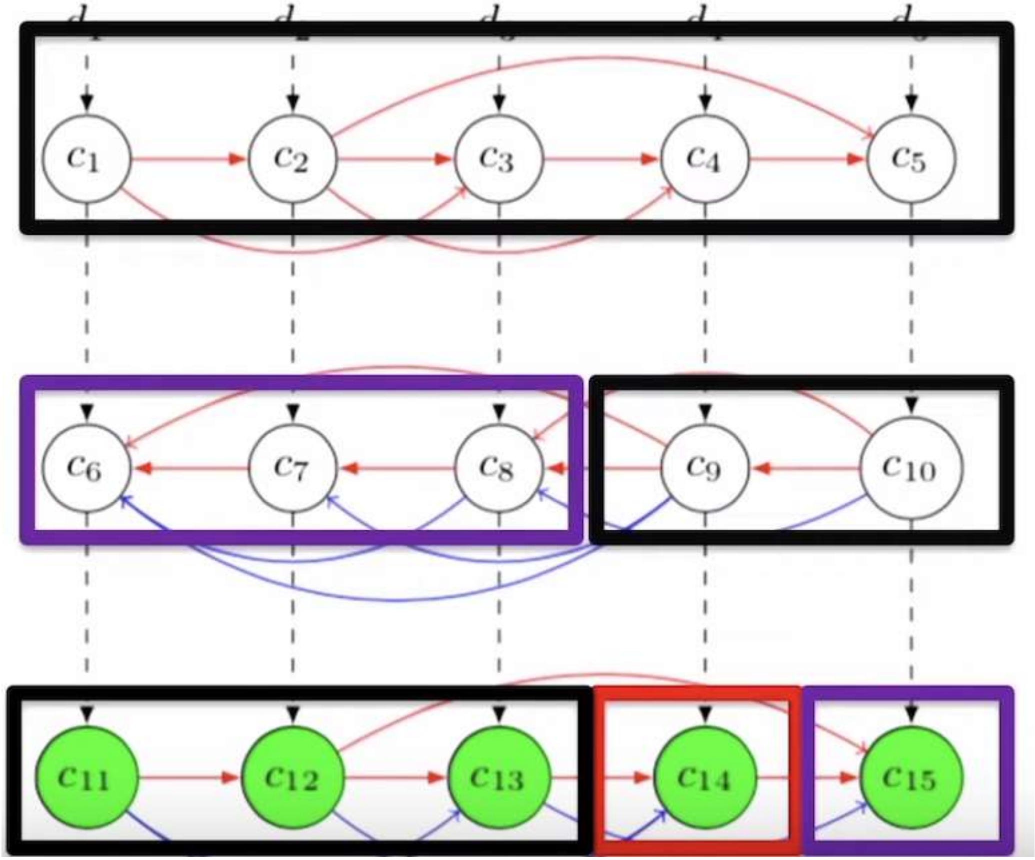
Check F against digest by hashing

Space requirement ✗

“Usefulness” requirement ✓

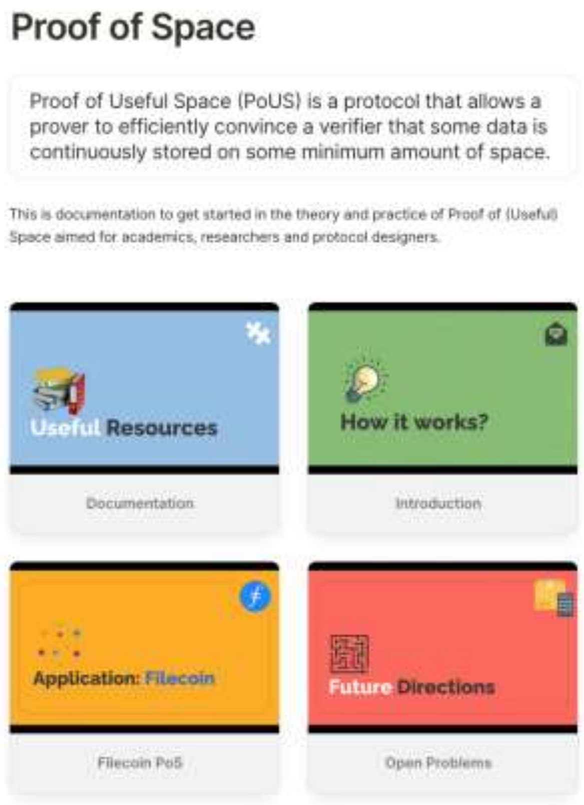
Security intuition: *“If prf checks then Prover is storing a certain amount of storage related to F”*

Thea naïve solution does not work.
So how do we solve these problems??



More resources and wrap up

- https://proofofspace.org/references
- Filecoin.io



Proof of Space

Proof of Useful Space (PoUS) is a protocol that allows a prover to efficiently convince a verifier that some data is continuously stored on some minimum amount of space.

This is documentation to get started in the theory and practice of Proof of (Useful) Space aimed for academics, researchers and protocol designers.

Useful Resources

Documentation

How it works?

Introduction

Application: Filecoin

Filecoin PoS

Future Directions

Open Problems

For any other questions:
matteo@protocol.ai

binarywhales.org

Thanks!